

One reason for adopting the new zero was that negative numbers and measurements in dealing with tide-gauge records, or soundings in surveys, would be thus avoided.

It is to be regretted that no precise references of mean tide level in the earlier days can be found. Where measurements do exist they are lacking in accurate information as to state of tides, and I can find nothing trustworthy upon which to base any statement as to change of sea-level since surveys have been made. I think it desirable that permanent bench marks on the natural beds or faces of rocks *in situ* should be established around our bay, carefully connected by accurate levelling with one another and with the tide-gauge, for it is very doubtful if bench marks on buildings can be assumed to afford a permanent datum.

---

ART. VIII.—*On the Method of Calculating the Increment  
in the Value of Land.*

BY ALEXANDER SUTHERLAND, M.A.

[Read 14th August, 1879.]

It ought to be well enough known in these enlightened times that a sum of money invested for a long term of years at a fair rate of interest, without being disturbed, will be converted into a sum quite enormous compared with the original investment; and yet the public mind is from time to time disturbed by fallacies originating in a comparison of first sum and its result without allowing for the action of compound interest. A man, for instance, may have spent £100 in buying a piece of land fifty years ago; he may have done nothing with it, and yet the land may now be worth the handsome little fortune of £11,731. When people learn this fact they are surprised beyond measure, and imagine the case quite exceptional; they fancy the man has got an enormous profit, and some of his poorer neighbours look upon the process as almost dishonest; whereas, as a matter of fact, he has got only 10 per cent. per annum on his investment. The present form that this fallacy takes is the demand that Government should retain possession of the

land of the colony and let it on long leases. Whether it should do so or not is a political question that we have nothing to do with. There seem to me to be many reasons why it might do so with advantage; but in this note I only seek to call your attention to an obvious mathematical fact that is commonly ignored in a most unaccountable manner.

In reckoning the rate at which the value of land increases, we must always adopt the methods of compound interest. No one pretends that, if a piece of land during the present year increases in value, this increment is at the end of the year taken out of the land; the increased value of the land has to remain throughout the next year as the capital invested, and the rate must be calculated upon that. To calculate it every year upon the original price of the land would be absurd. And yet this is what has been done by a recent writer in the *Melbourne Review*, who seeks to prove that the value of the land in Melbourne has increased at the rate of 139 per cent. per annum. If this statement is put forward by a well-informed writer, we may guess what misconception prevails generally. If £100 had originally been spent in the purchase of land, and it had increased in this rate, the value in the second year would be £239, for none of the increased value has been taken out of it. Now, if it still increases at the rate of 139 per cent. per annum, this must be reckoned on the £239, and not on the original £100. To show how utterly astray the calculation is, I need only mention that at this rate, even assuming so short a period as twenty-one years, the present value of the land of Melbourne would be £24,190,000,000,000 instead of its real value, £8,286,000.

I have no doubt that to any one conversant with mathematics it will seem a very elementary proposition that the method of compound interest is the only one allowable in such a case, but I should like to have the matter discussed before our society, so that the true method to be adopted may be settled, and possibly our legislators prevented from rushing into hasty measures grounded on fallacious reasons.

The question of rents has come in as a disturbing element. Not only here, but in England and France, it has been tacitly assumed that, as a man gets from year to year, in the shape of rent, what is reckoned as a fair interest, the increment can then be calculated as at simple interest. But I need hardly point out to a mathematician how faulty such a process would be. The only correct method is to calculate

as compound interest the total increment, and then add in the yearly rate per cent. given by the rents. Thus £100 worth of land increasing at the rate of 10 per cent., and producing a rental of 5 per cent., will give these figures:—

First Year	.....Value £100.....	Rent £5
Second Year.....	Value £110.....	Rent £5 10s.
Third Year	.....Value £121.....	Rent £6 1s.

And so on. Now take the third year, when the value of the land is increasing by the sum of £12 2s., while for rent it produces £6 1s. The total yield of the land is £18 3s., and yet if this be calculated as interest on £121, which is the value of the land, and which the owner could have got for his land if he chose, it will give only 15 per cent., and this is the sum of 10 per cent. increment and 5 per cent. interest.

The calculation of the exact rate at which the value of the land in Victoria has increased would involve the solution of an equation of the forty-second degree, and the labour so involved would not be compensated by the degree of accuracy obtained.

The following equation will give the rate of increase in the value in Victoria—

$$\begin{array}{r}
 330,629 R^{40} + \quad \quad 77,213 R^{35} + \quad \quad 116,935 R^{30} + \\
 1,453,383 R^{25} + \quad 2,144,999 R^{20} + \quad 2,054,839 R^{15} + \\
 1,689,181 R^{10} + \quad 2,610,085 R^5 = V
 \end{array}$$

where  $V$  is the value of all the land in the colony.

In this equation, in order to reduce the terms from forty in number down to eight, I have grouped the land sales together for periods of five years, and taken the middle of such periods as the average time of the sales.

A difficulty arises in determining  $V$ . The year-book of Mr. Hayter gives £65,000,000 as the total rateable property of the colony, but this includes all the buildings in cities, townships, or farms, together with all improvements on farm or pastoral lands, and to get the value of the land by itself seems impossible.

If the rate per cent. be taken as 5, then the above equation will give  $R=1.05$ , and the value of  $V$  is £24,000,000, which is perhaps under the mark, but whenever the real value of  $V$  is attained some such equation as the above must be used in order to give tolerably accurate results.

About 6 per cent., I think, might be found to be the real value of the rate of increment. To this add the rate received from rents, and the total will be the interest obtained by the aggregate of landholders in Victoria.

More definite results can be obtained when we take more confined areas. The most startling results are generally adduced from land in Melbourne. The complete returns of land sales in Melbourne proper I have not been able to obtain, but the following results will give something like the correct rates. In the case of the sites in the very heart of the city, the calculation can best be effected by taking the average value of the land sold in 1837 and comparing that with the average value at present. The average price per acre paid in 1837 was £70, the average value at present is about £25,000 for the central portion. This seems an enormous rate of increase, but it is not so in reality.

$$70 R^{42} = 25,000$$

$$\text{Log. } 70 + 42 \text{ log. } R = \text{log. } 25,000$$

$$1.8450980 + 42 \text{ log. } R = 4.3979400$$

$$42 \text{ log. } R = 2.5528420$$

$$\text{log. } R = .060782$$

$$R = 1.152$$

Hence the rate per cent. is 15.

But those who bought land here in 1837 obtained it under exceptional circumstances. It was they themselves that gave the land any value whatever. Take the case of those who bought land in 1840. The average price was then £842 per acre. Its average price is now £25,000. Hence—

$$842 R^{39} = 25,000$$

$$39 \text{ log. } R = \text{log. } 25,000 - \text{log. } 842$$

$$= 4.3979400 - 2.9253121$$

$$= 1.4726279$$

$$\text{log. } R = .037751$$

$$R = 1.0908$$

$$R - 1 = .0908$$

Therefore the rate is 9 per cent. per annum. To this add the rate received for rents, and about 13 or 14 per cent. is the real rate received by the landholder.

The proposition I should like to bring forward for your consideration is this:—

If  $r^1$  be the rate per cent. per annum yielded by the rent of a piece of land, and  $r$  be the rate per cent. per annum of the increment in its value—

If  $V^1$  be the price originally paid for land,  $V_2$  the value after  $n$  years, then the value of  $r$  is obtained from the equation—

$$\log. (1 + r) = \frac{\log. V_2 - \log. V_1}{n}$$

If  $R$  be value of  $r$  obtained from this equation, then  $R + r$  is the annual rate per cent. received by the investor on his money.

ART. IX.—*Hughes' Induction Currents Balance and Sonometer.*

BY ROBERT E. JOSEPH.

[Read 14th August, 1879.]

ON the 15th of May last Professor Hughes read before the Royal Society, in London, a paper on "An Induction Currents Balance, and Experimental Researches made with it;" and the subject appeared to me to be of so much interest, and capable of being utilised to a very great extent in physical research, that I considered a few notes on the matter would prove acceptable to the members of our own Society.

Attempts have frequently been made to construct induction-balances for many years past; but they do not appear to have given satisfactory results. The introduction of the telephone, however, as an instrument that will detect currents of too feeble a nature to be detected by any other instrument, has caused what before was found to be a difficulty to now become quite easy, besides being simple and reliable.

The instruments I introduce to your notice this evening are the induction-balance, sonometer, clock-microphone, and telephone, which, with a battery and reversing key, will enable us to make any experiments we may require. Before, however, proceeding with these experiments, it will be as